Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2015**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

1 In the expansion of $\left(1 - \frac{2x}{a}\right)(a + x)^5$, where a is a non-zero constant, show that the coefficient of x^2 is zero. [3]

2 The function f is such that $f'(x) = 3x^2 - 7$ and $f(3) = 5$. Find $f(x)$. [3]

3 Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

4 **(i)** Show that the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$ [3]

(ii) Hence solve the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

5 A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

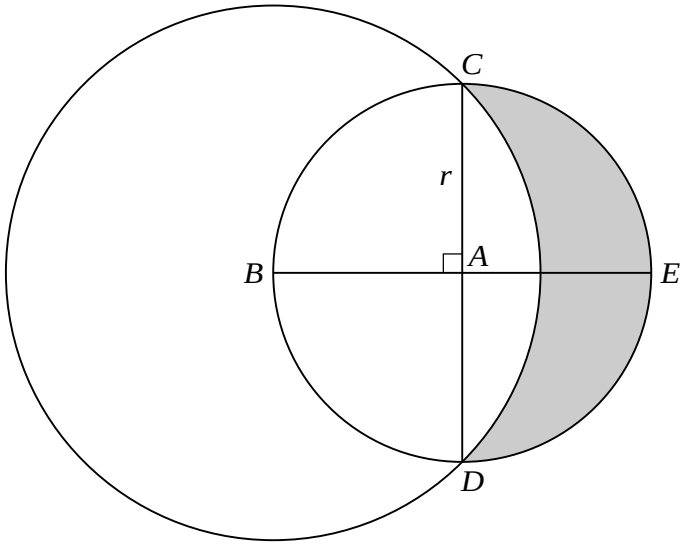
6 A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

7



The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(ii) Find the area of the shaded region in terms of r. [6]

8 The first term of a progression is $4x$ and the second term is x^2.

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of x and the corresponding values of the third term. [4]

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term. [4]

9 **(i)** Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

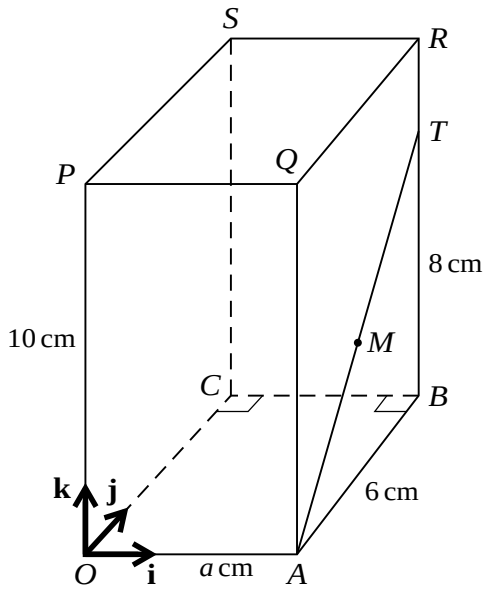
The function $\mathrm{f} : x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

(ii) State the smallest value of m for which f is one-one. [1]

(iii) For the case where $m = 5$, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

[Questions 10 and 11 are printed on the next page.]

10

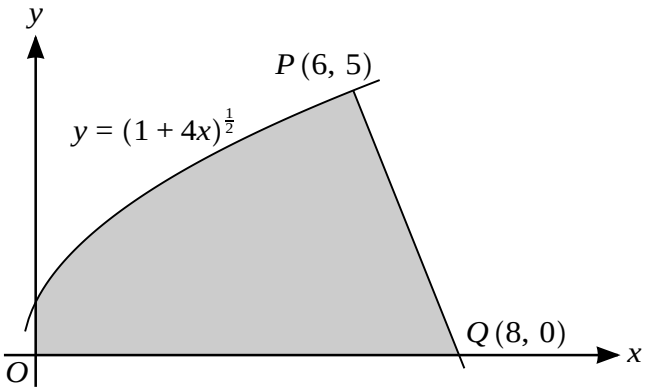


The diagram shows a cuboid $OABCPQRS$ with a horizontal base $OABC$ in which $AB = 6$ cm and $OA = a$ cm, where a is a constant. The height OP of the cuboid is 10 cm. The point T on BR is such that $BT = 8$ cm, and M is the mid-point of AT. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OP respectively.

(i) For the case where $a = 2$, find the unit vector in the direction of $\overrightarrow{PM}$. [4]

(ii) For the case where angle $ATP = \cos^{-1}\left(\frac{2}{7}\right)$, find the value of a. [5]

11



The diagram shows part of the curve $y = (1 + 4x)^{\frac{1}{2}}$ and a point $P\,(6,\ 5)$ lying on the curve. The line PQ intersects the x-axis at $Q\,(8,\ 0)$.

(i) Show that PQ is a normal to the curve. [5]

(ii) Find, showing all necessary working, the exact volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [7]

[In part **(ii)** you may find it useful to apply the fact that the volume, V, of a cone of base radius r and vertical height h, is given by $V = \frac{1}{3}\pi r^2 h$.]